1. Solve the following recurrences.
   (a) $T(n) = 2T(n/2) + n$ using both Master theorem and the recursion tree method.
   (b) $T(n) = T(n/2) + O(n)$ by expanding out the recurrence.
   (c) $T(n) = T(n-a) + T(a) + n$, where $a \geq 1$ is a constant.
2. The running time of an algorithm $A$ is described by the recurrence $T(n) = 4T(n/2) + n^2$. A competing algorithm $A'$ has a running time of $T'(n) = aT'(n/4) + n^2$. What is the largest integer value for $a$ such that $A'$ is asymptotically faster than $A$?
3. Consider the regularity condition $af(n/b) \leq cf(n)$ for some constant $c < 1$, which is part of case 3 of Master theorem. Give an example of a simple function $f(n)$ that satisfies all the conditions in case 3 of Master theorem except the regularity condition.
4. It is known that $a > 0$. Find all possible values of $a$ if
   (a) $T(n) = \Theta(n^2 \log n)$ is a solution to $T(n) = aT(n/2) + \Theta(n^2)$;
   (b) $T(n) = \Theta(n^2)$ is a solution to $T(n) = aT(n/3) + \Theta(n)$;
   (c) $T(n) = \Theta(n^2)$ is a solution to $T(n) = 4T(n/a) + \Theta(n^2)$.
5. **(Majority element)** Develop a recursive algorithm that finds the majority element in an array of size $n$. The majority element is the element that appears more than $\lfloor n/2 \rfloor$ times. (You can assume that such an element exists.)
6. **(The skyline problem)** A city's skyline is the outer contour of the silhouette formed by all the buildings in that city when viewed from a distance. Now suppose you are given the locations and height of all the buildings. Develop a recursive algorithm that outputs the skyline formed by these buildings collectively.
7. **(Tower of Hanoi)** The Tower of Hanoi puzzle was invented by the French mathematician Edouard Lucas in 1883. He was inspired by a legend that tells of a Hindu temple where the puzzle was presented to young priests. At the beginning of time, the priests were given three poles and a stack of $n = 64$ gold disks, each disk a little smaller than the one beneath it. Their assignment was to transfer all $n$ disks from one of the three poles to another, with two important constraints. They could only move one disk at a time, and they could never place a larger disk on top of a smaller one. The priests worked very efficiently, day and night, moving one disk every second. When they finished their work, the legend said, the temple would crumble into dust and the world would vanish. Develop a recursive algorithm for a general value of $n$. Give asymptotic upper and lower bounds for its running time. Make your bounds as tight as possible, and justify your answers.
8. **(Hamming distance)** Hamming distance between two strings of equal length is defined as the number of mismatches between them. For example, the Hamming distance between "0010" and "1010" is one. Given a binary string $T$ of length $2n$ (text) and a binary string of length $n$ (pattern), develop an algorithm that computes the Hamming distance between each $n$-length substring of $T$ and $P$.